Algebra II – Lesson 3: Linear Equations (page 1)
Upward Bound Summer 2018 **Name: ______________________**

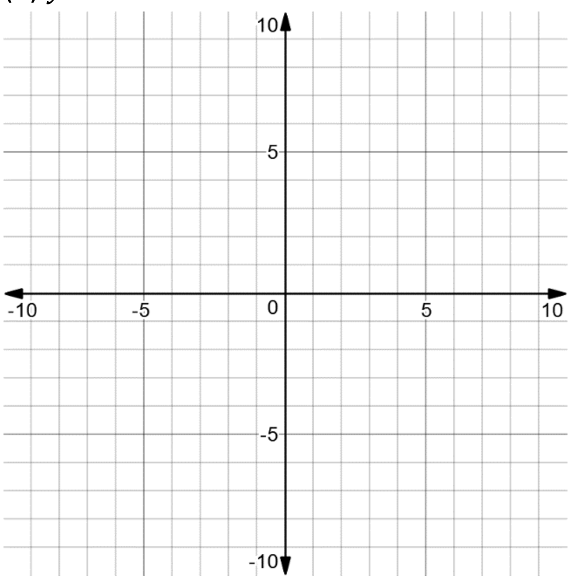
<u>Basic: What is a linear equation?</u>

A **linear equation** is an equation which can be written in the form $y = mx + b$, where m and b are integers.

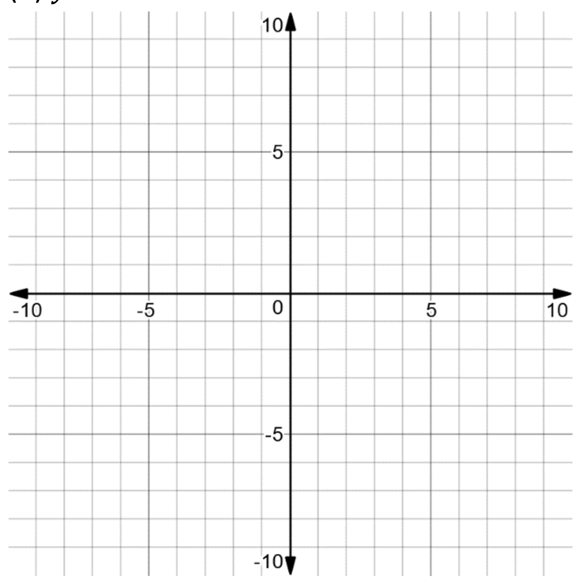
- The **slope** of a linear equation is m. Every time you go right 1 unit on the graph, you go up m units. If m is a fraction, you can view it as $\frac{rise}{run}$. So if $m = \frac{2}{3}$, then every time you go right 3 units on the graph (which is the "run"), you go up 2 units on the graph (which is the "rise").
- The **y-intercept** (or **vertical intercept** if your dependent variable is something other than y) is b. The graph "starts at" b on the y-axis. When $x = 0$, $y = b$.

Graph each of the following linear equations:

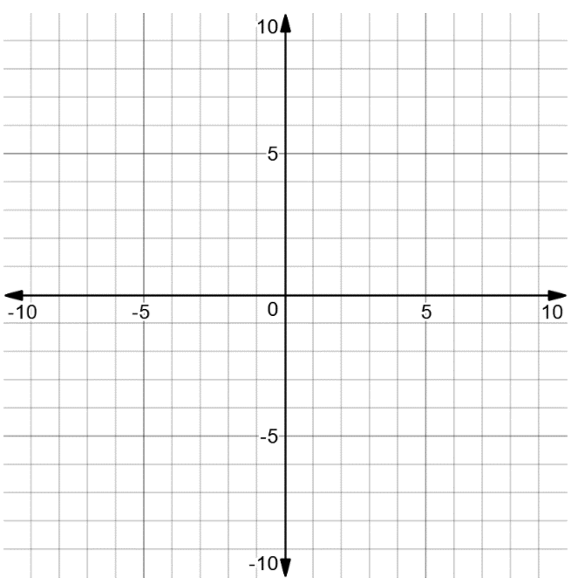
(1) $y = x$



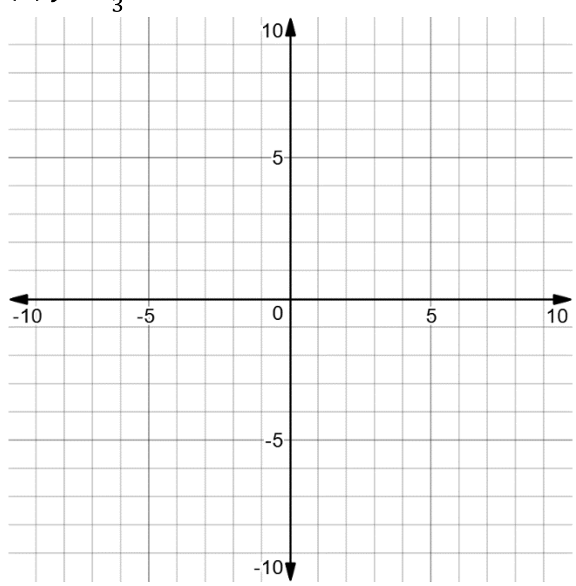
(2) $y = 2x$



(3) $y = x + 2$



(4) $y = \frac{2}{3}x - 1$



Algebra II – Lesson 3: Linear Equations (page 2)
Upward Bound Summer 2018 **Name: ____________________**

You can find a linear equation from two given points by using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and then either plugging in a point to solve for b or plugging in a point to get point-slope form.

- In the slope formula, (x_1, y_1) and (x_2, y_2) represent two points on the line.
- We mostly use **Slope-Intercept Form** $y = mx + b$, but there are also two other ways people commonly write linear equations:
  - **Point-Slope Form** $y = y_1 + m(x - x_1)$ allows us to get an equation by plugging in a point (x_1, y_1) and the slope m. Since a line contains many points, there are many different ways you could write the same equation in point-slope form. However, if you then simplify any of them, you will get the unique way to write the equation in slope-intercept form.
  - **Standard Form** $Ax + By = C$ makes it so you can solve for the x- and y-intercepts equally easily (which you could then connect to graph the line), and it will be useful for one of our methods of solving systems of equations in the next lesson. Here, we prefer that A, B, and C are integers with no common factor other than 1, and we prefer A to be positive.

For each of the following pairs of points on a line, write the linear equation in all 3 forms:

Prob. #	Point 1	Point 2	Slope-Intercept	Point-Slope	Standard
5	$(0,0)$	$(1,3)$			
6	$(0,4)$	$(2,6)$			
7	$(-6,0)$	$(0,-2)$			
8	$(2,4)$	$(7,4)$			
9	$(3,5)$	$(6,3)$			

Notice that Problem #8 was a horizontal line. Horizontal lines have equation $y = k$ for some number k, but vertical lines are not functions because they do not pass the vertical line test. They have equations of the form $x = h$ for some number h.

(10) Write the equation of a horizontal line passing through the point $(4, -1)$.

(11) Write the equation of a vertical line passing through the point $(4, -1)$.

Algebra II – Lesson 3: Linear Equations (page 3)
Upward Bound Summer 2018 **Name: ______________________**

Intermediate: Parallel and Perpendicular

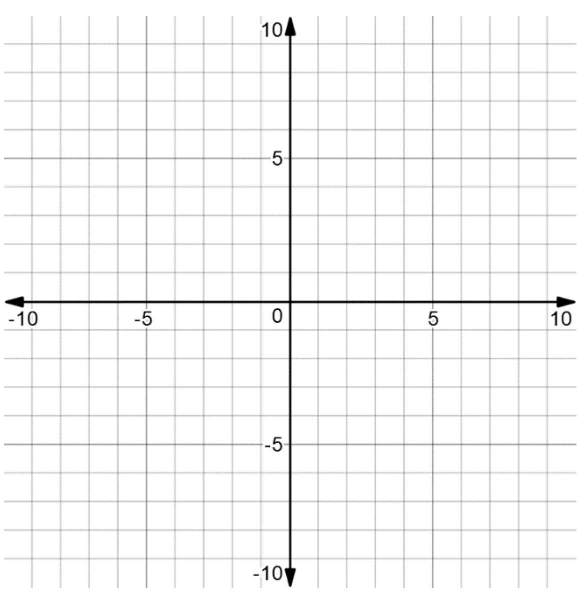
If two lines are **parallel**, then they never intersect. This means they have the same slope: $m_1 = m_2$

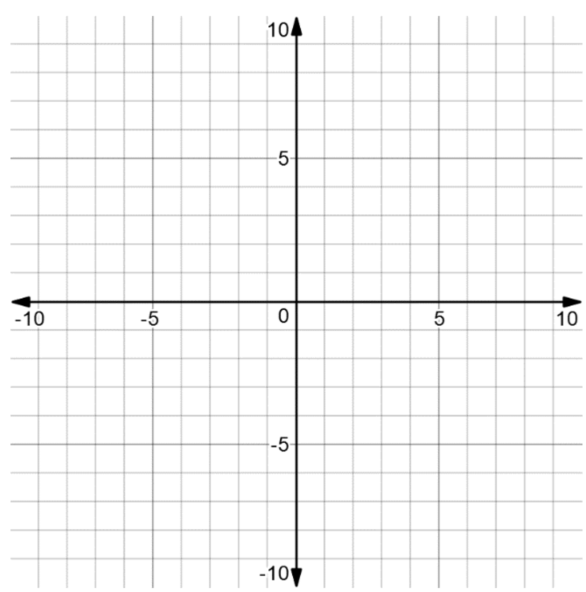
If two lines are **perpendicular**, then they intersect at a right angle. This means their slopes are negative reciprocals of each other: $m_1 = -\frac{1}{m_2}$

For each of the following pairs of a line and a point, write the equation of the line parallel to the given line which passes through the given point. Then do the same for a perpendicular line.

Prob. #	Line	Point	Parallel Line	Perpendicular Line
12	$y = 2x$	$(0,9)$		
13	$y = 3 - x$	$(3,-2)$		
14	$y = -\frac{1}{2}x - 2$	$(5,4)$		
15	$y = \frac{4}{3}x + 6$	$(-4,0)$		
16	$y = 7$	$(1,-1)$		

(17) On the left, graph the given line and parallel line from Problem #13 on the same set of axes.
(18) On the right, graph the given line and perpendicular line from Problem #14 on the same set of axes.





Algebra II – Lesson 3: Linear Equations (page 4)
Upward Bound Summer 2018 **Name: ____________________**

Advanced: Multivariable Linear Equations

We don't need to use just x and y for a linear equation.

One thing we could do is change our variable names. We could say, for example $D = 20t$ if we wanted our dependent variable to be D and our independent variable to be t.

Another thing we could do is have multiple variables. If the cost of a child movie ticket is $10 and the cost of an adult movie ticket is $14, you might use the multivariable linear equation $R = 10c + 14a$ to find the revenue of the theater (which depends on both the number of child and number of adult tickets sold).

The equation $y = ax^2$ is clearly not linear in x (because of the squaring), but it is linear in a (because there are no powers of a involved).

In general, a multivariable linear equation looks like $y = b_0 + b_1x_1 + b_2x_2 + \cdots + b_nx_n$.

(19) Consider the following situation:

You are producing and selling fidget spinners. Each spinner costs you $1 to produce, but you can sell them for $2.50 each. Let P represent your profit, m represent the number of fidget spinners you produce, and n represent the number of fidget spinners you sell. Write a multivariable linear equation for your profit.

(20) If you sell every fidget spinner you produce, then what is your profit per fidget spinner?

For problems #21-22, suppose you produce 50% more fidget spinners than you manage to sell.

(21) Write an equation for m in terms of n.

(22) Write a linear equation for the profit P in terms of only n (instead of m and n).